Resonance

by Larry Luchi W7KZE

[Editor's Note: This technical discussion of resonance is a bit more advanced than what we usually publish in Radio Fun, *and it may leave some of you in the dark. If so, read it through a few times—slowly—and things should begin to sink in. If you want to get the most out of amateur radio, you've got to really understand this stuff—not just memorize a bunch of answers for a test. We've asked Mr. Luchi to prepare a series of articles focusing on various electronic components and their properties. We'll publish them from time to time, and if you stick with them, you'll be well on your way to a solid foundation in electronics.—David N1GPH]*

Resonant circuits are at the heart of all transmitters, receivers, and antennas. Without resonant circuits there would be no radio communication.

Brief mention was made of resonance in my last article. When the inductive reactance (X_L) of a coil equals the capacitive reactance (X_C) of a capacitor in a circuit, a condition known as resonance occurs. Figure 1a illustrates a series-resonant circuit and Figure 1b shows a parallel-resonant circuit.

Since resonance is the condition where X_L equals X_C, the formula for resonance is:

$$X_L = X_C \text{ or } 2\pi fL = \frac{1}{2\pi fC} \quad \text{Equation 1}$$

Where X_L = inductive reactance, in ohms

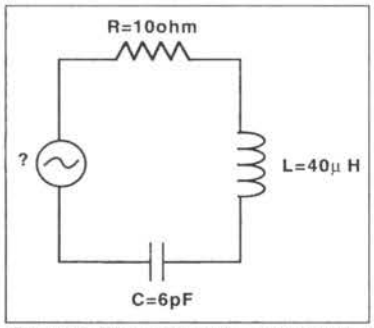

Figure 2. The voltage drop across the coil and the capacitor is zero. The total source voltage appears across R.

X_C = capacitive reactance, in ohms
f = frequency, in hertz
L = inductance, in henries
C = capacitance in farads

Equation 1 shows that the inductive reactance is directly proportional to frequency, and that capacitive reactance is inversely proportional to frequency. With any given coil and capacitor, as the frequency increases the reactance of the coil increases but the reactance of the capacitor decreases. At some frequency, the two reactances will be equal in value. At this exact frequency resonance occurs. At all other frequencies the circuit shown in Figure 1a is merely a series AC circuit, and Figure 1b is just a simple parallel AC circuit.

To find the frequency at which a coil and capacitor will resonate, the resonance formula may be rearranged to solve for f:

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad \text{Equation 2}$$

This formula can be used to find the resonant frequency of any LC circuit. For example, if we have an X_L of 1,070 ohms and an X_C of 1,068 ohms, we can see that L and C are at a resonant frequency. [$X_L = X_C$ (close enough)]Using the above formula with an inductor of 8 μH and capacitance of 7 pF, we can solve for the resonant frequency.

When using Equation 2 with a calculator, use the [EE] or [EXP] key for the exponent (power of 10). Your calculator will then default to a positive exponent. The sign change key [+/-] will change to a negative exponent for millihenries, 10^{-3} or microhenries, (10^{-6}) or picofarad (10^{-12}). [ENG] key changes any exponent to a metric value, which is to say any exponent that can be divided by three. All of the above exponents are [ENG]. Practice with the following example:

Example 1:

ENTER	DISPLAY
[CE/C]	0
8 [EXP] [+/-] 6 [X]	0.000008
7 [EXP] [+/-] 12 [=]	5.6^{-17}
[2nd] [√]	0.000000007
[X] [6.28] [=]	0.0000000046
[1/X]	21278761.3
[2nd] or [EE]	21.3^{6} or 21.3 MHz

According to the formula, the frequency at resonance is inversely proportional to the square root of either L or C. Any time the LC product is decreased to one-quarter by any means the frequency will be doubled.

Any time the LC product is increased four times—by quadrupling L, by quadrupling C, by doubling both L and C, or by any other means—the frequency will be one-half of the original.

Series Resonant Circuits

In any series circuit containing both L and C, the circuit current is greatest when the inductive reactance X_L equals the capacitive reactance X_C. Because, under those conditions, the impedance is equal to R. When resonance occurs, the reactances are equal, and we can derive an equation to calculate the resonant frequency of any inductor-capacitor pair using Equation 2.

If the source sees the coil and capacitor together as having a zero voltage-drop across them, it sees them as perfect conductors, or as having zero impedance (Figure 2).

If the reactances are not exactly equal (a nonresonant condition), the voltages do not exactly cancel. The source sees the two reactances as having a resultant voltage-drop across them and therefore as having some value of reactance or impedance.

Theoretically, if a series LC circuit has no resistance, and is connected across a source of AC to which it is resonant, it presents zero reactance, zero resistance, zero impedance, and infinite current!

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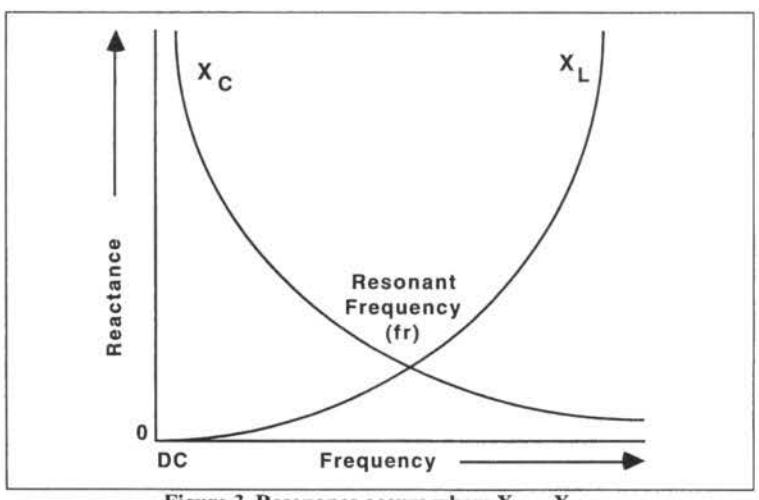

Figure 3. Resonance occurs where $X_C = X_L$.

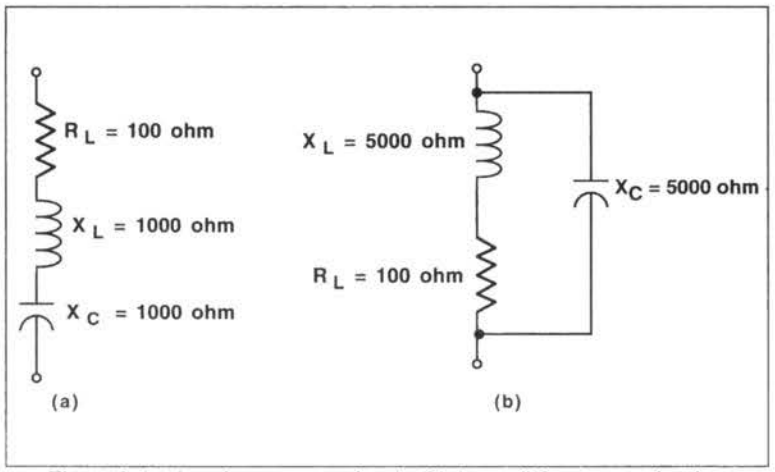

Figure 1. (a) A series-resonant circuit. (b) A parallel-resonant circuit.

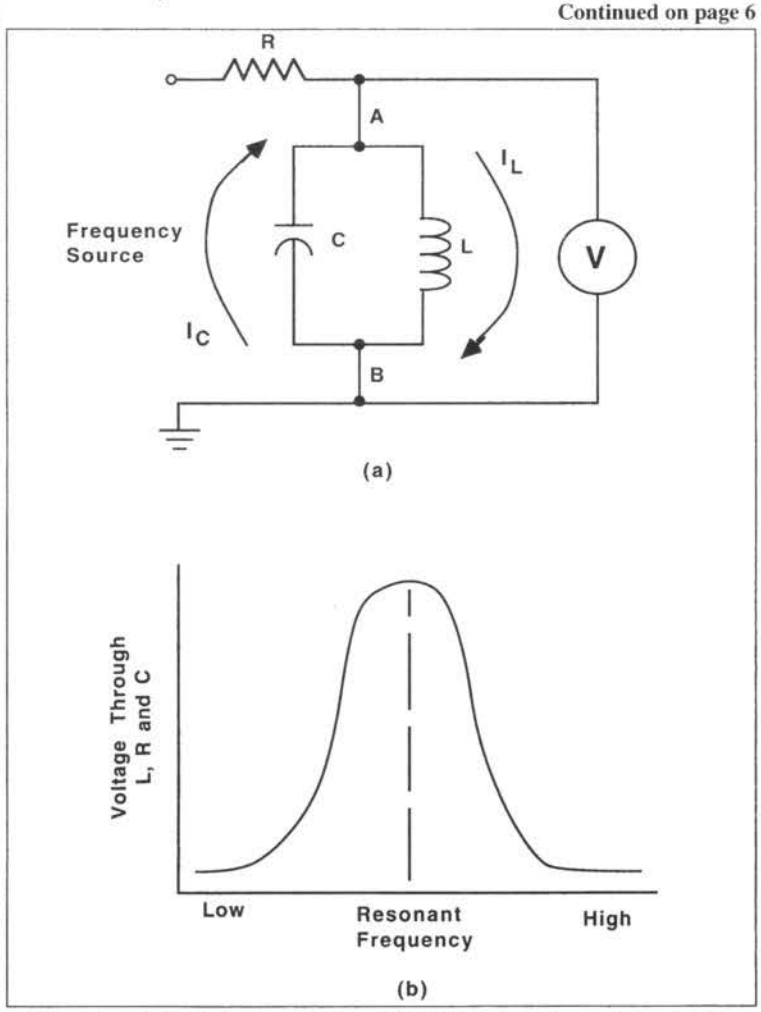

Figure 4. (a) Parallel-resonant circuit. (b) *Voltage maximum* at resonance.

Resonance

Continued from page 4

Parallel Resonant Circuits

Resonance in parallel circuits occurs when $X_L = X_C$, as in series circuit resonance. In Figure 3, for a specific inductor and a specific capacitor, the point where the two curves cross ($X_L = X_C$) is called resonant frequency. When a parallel circuit has equal X_L and X_C, the external circuit current is equal to that flowing through the parallel resistance. If the circuit contains no parallel resistance, the external current is zero. A coil and capacitor connected as in Figure 4a form a parallel AC circuit. If X_L and X_C have the same reactance to the frequency of the AC source, the circuit is known as parallel resonant. In Figure 4a let X_C equal 100 ohms and X_L equal 100 ohms; with a source voltage of 100 volts we can calculate the current.

If the capacitor is temporarily disconnected, leaving the 100Ω reactance coil across the 100 V source, according to Ohm's law the AC circuit current in the coil will be:

$$I = \frac{V}{X_L} = 1 \text{ amp}$$

If the 100Ω reactance capacitor is reconnected across the coil, 1 amp of current will flow in the capacitor. In a parallel resonant circuit you read equal voltage across the coil and the capacitor. In the inductive branch, the current lags the source voltage by 90 degrees, and in the capacitive branch the current leads the source voltage by 90 degrees. Since the two currents are 180 degrees out of phase, at any instant that current is flowing down through the coil and equal current must be flowing up into the capacitor.

This circulating current flows between the two reactances, but no current flows into and out of the source. Because the source is supplying no current, it should be possible to disconnect the source and the current should continue to oscillate back and forth between the capacitor and the inductor in Figure 4a indefinitely. With no resistance or losses in the circuit, this would be true. This ability of a resonant circuit to sustain electron oscillation is known as the flywheel effect because of its similarity to the action of a mechanical flywheel, which, once started, tends to keep going until stopped by friction or other losses.

Calculating the resonant frequency of a parallel circuit is exactly the same as for a series resonance circuit.

Remember, in Figure 3, for a specific inductor and a specific capacitor, the point where the two curves cross ($X_L = X_C$) is called resonant frequency. With pure reactance and zero resistance in the circuit, the source current is zero. With resistance in series with either branch, source current increases, and reactive currents decrease. That is to say, at resonance parallel circuit voltage is at maximum!

Calculating Component Values for Resonance

We can calculate either the inductance L or capacitance C to resonate with a certain component at a specific frequency:

$$L = \frac{1}{(2\pi f_r)^2 C}$$

Example 1: What value of L resonates with a 106 pF capacitor at 1,000 kHz?

ENTER	DISPLAY
[CE/C]	0
[(] [6.28] [X] 1 [EXP] 3 [)] [X^2]	39438400
[X] 106 [EXP] [+/-] 12 [=]	0.00418047
[1/X]	239.2075303 or 239Ω

Example 2: What value of L resonates with a 200 pF capacitor at 7,200 kHz?

ENTER	DISPLAY
[CE/C]	0
[(] [6.28] [X] 7.2 [EXP] 6 [)] [X2]	2.044486656^{15}
[X] 200 [EXP] [+/-] 12 [=]	408897.3312
[1/X]	0.000002445
[2nd] [ENG]	2.44^{-06}

Example 3: What value of C resonates with a 239 µH inductor at 1,000 kHz?

ENTER	DISPLAY
[CE/C]	0
[(] [6.28] [X] 1 [EXP] 3 [)] [X^2]	39438400
[X] 239 [EXP] [+/-] 6 [=]	9425.7776
[1/X]	0.000106092
[2nd] [ENG]	106^{-6}

Finding L from X_L: Not only can X_L be calculated from f and L, but if any two factors are known, the third can be found. Very often X_L can be determined from voltage and current measurements. With the frequency known, L can be calculated as:

$$L = \frac{X_L}{2\pi f}$$

Example 4: A coil with negligible resistance has 62.8 volts across it with 0.01A of current. What is the inductive reactance X_L?

$$X_L = \frac{V_L}{I_L} = \frac{62.8V}{0.01A} = X_L = 6{,}280\Omega$$

Example 5: Calculate the L of an inductor that has a 15,700ΩX_L at a frequency of 10 MHz.

ENTER	DISPLAY
[CE/C]	0
[(] 15,700 [)] [÷] [(]	0
6.28 [X] 10 [EXP] 6 [)]	62,800,000
[=]	0.00025
[2nd] [Eng]	250^{-06}

If this calculation is performed without parentheses, the algebraic hierarchy completes the multiplication before the addition. Without parentheses the result is 2.5^{10}H.

In my next article I will discuss impedance and the Q of a circuit. RF

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